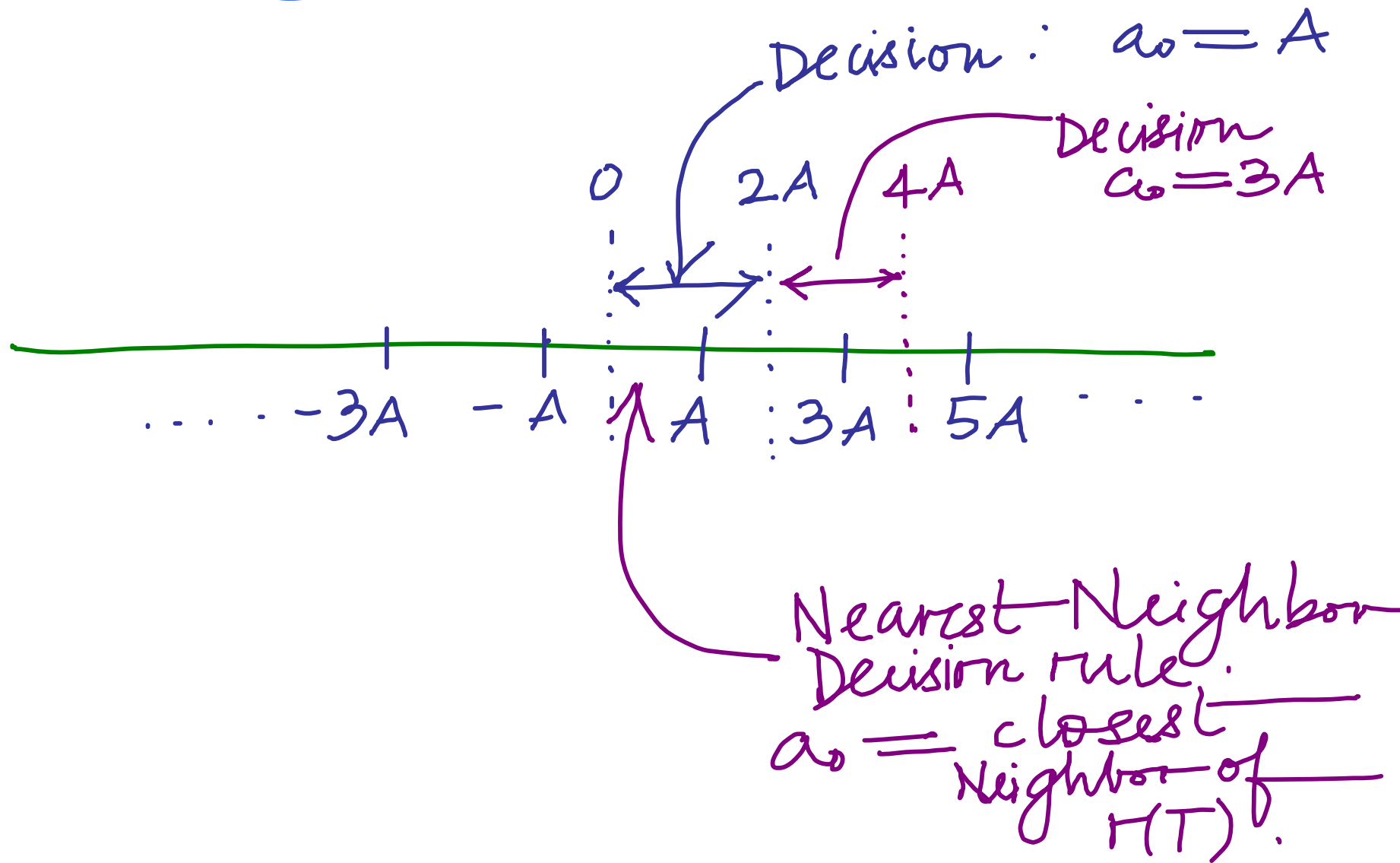
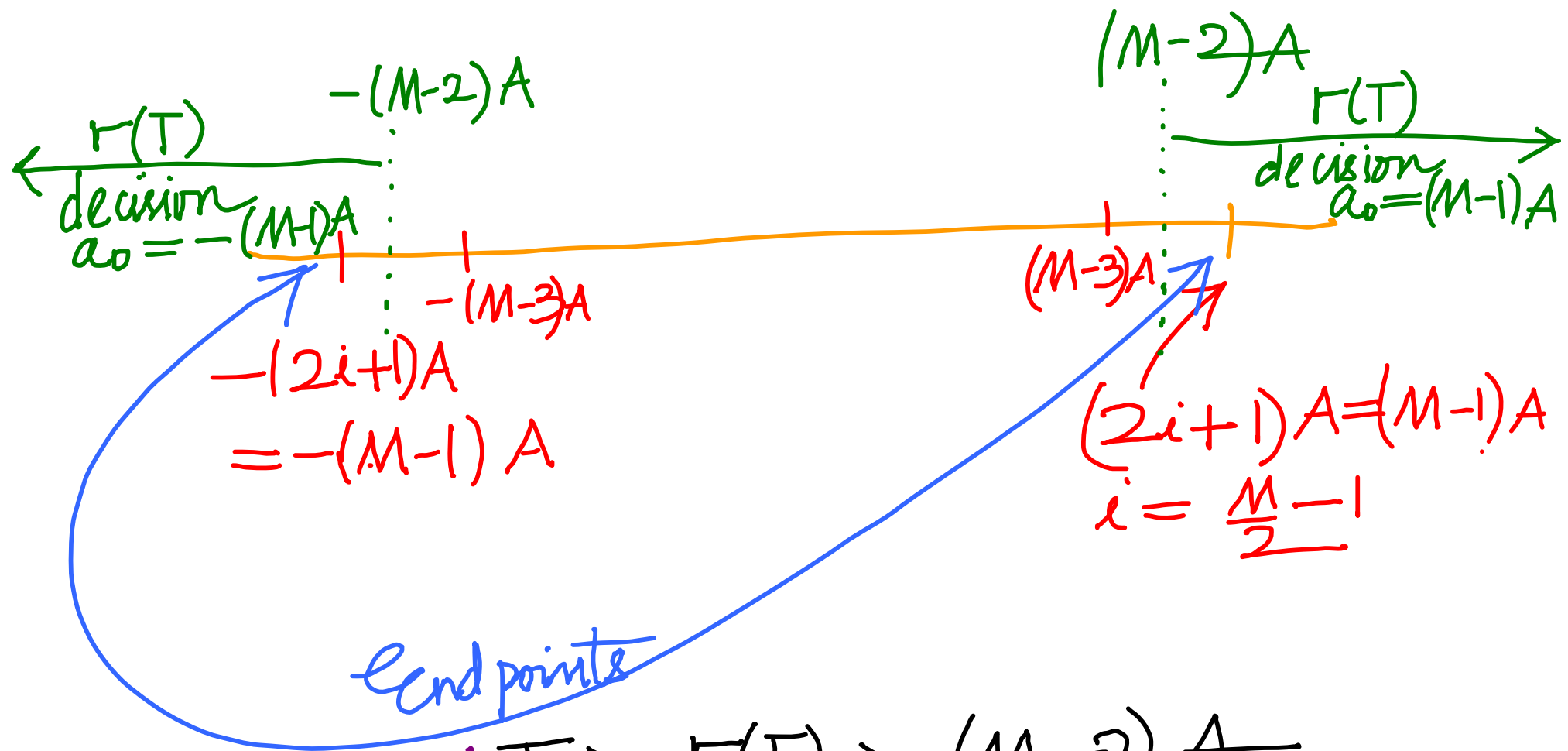


X

M-ary PAM:





$$\text{If } r(T) \geq (M-2)A$$

$$\text{Decision } a_0 = (M-1)A$$

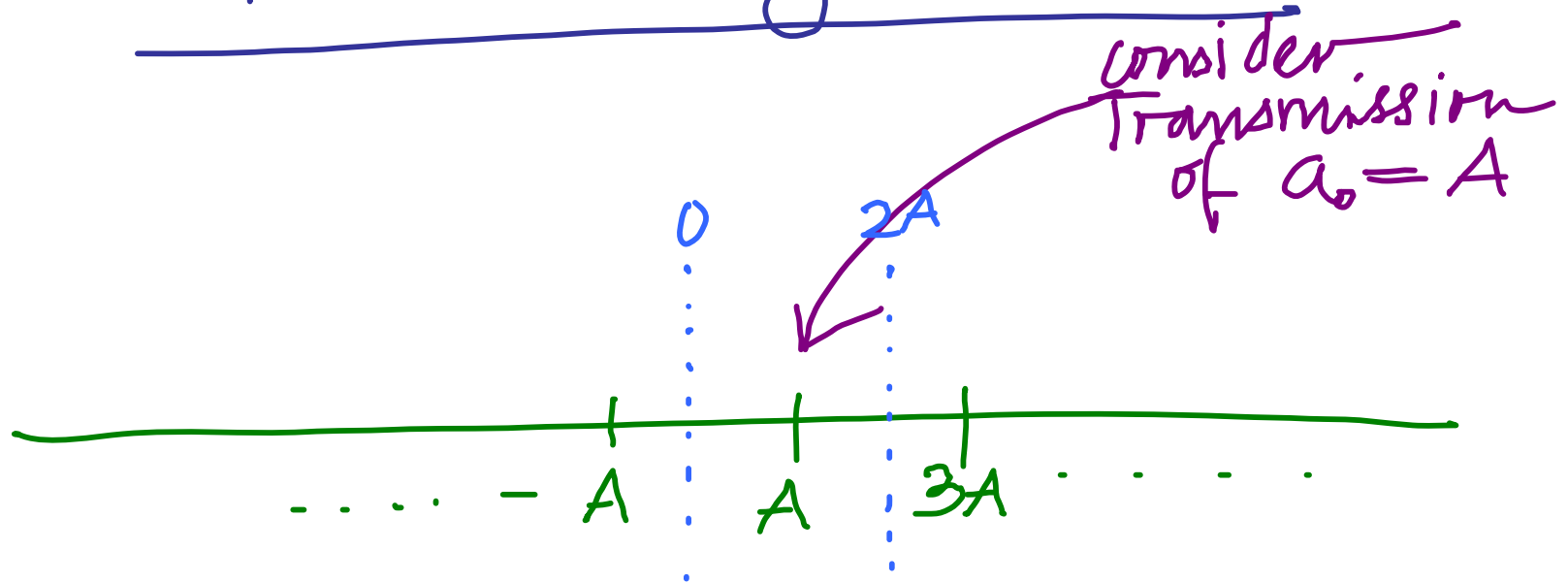
$$r(T) < -(M-2)A$$

$$\text{Decision } a_0 = -(M-1)A$$

Together with previous cases completes the set of decision rules.

Probability of Error

For M-ary PAM



decide $a_0 = A$ if

$$0 \leq r(T) < 2A$$

Error occurs if
either $r(T) \geq 2A$

Prob of error

$$\text{or } r(T) < 0.$$

$$Pr(e) = Pr(\overbrace{r(T) < 0}^{\phi_1} \cup \overbrace{r(T) \geq 2A}^{\phi_2})$$

$$P_e = Pr(r(T) < 0) + Pr(r(T) \geq 2A)$$

$$Pr(\phi_1 \cup \phi_2)$$

Disjoint Events
Mutually Exclusive

$$Pr(\phi_1 \cup \phi_2)$$

$$= Pr(\phi_1) + Pr(\phi_2)$$

$$r(T) = a_0 + \tilde{n}$$
$$= A + \tilde{n}$$

Gaussian
mean = 0
var = $\frac{N_e}{2}$

Error if $\Gamma(T) \geq 2A$

$$\Rightarrow A + \tilde{n} \geq 2A$$

$$\Rightarrow \boxed{\tilde{n} \geq A}$$

or $\Gamma(T) < 0$

$$\Rightarrow A + \tilde{n} < 0$$

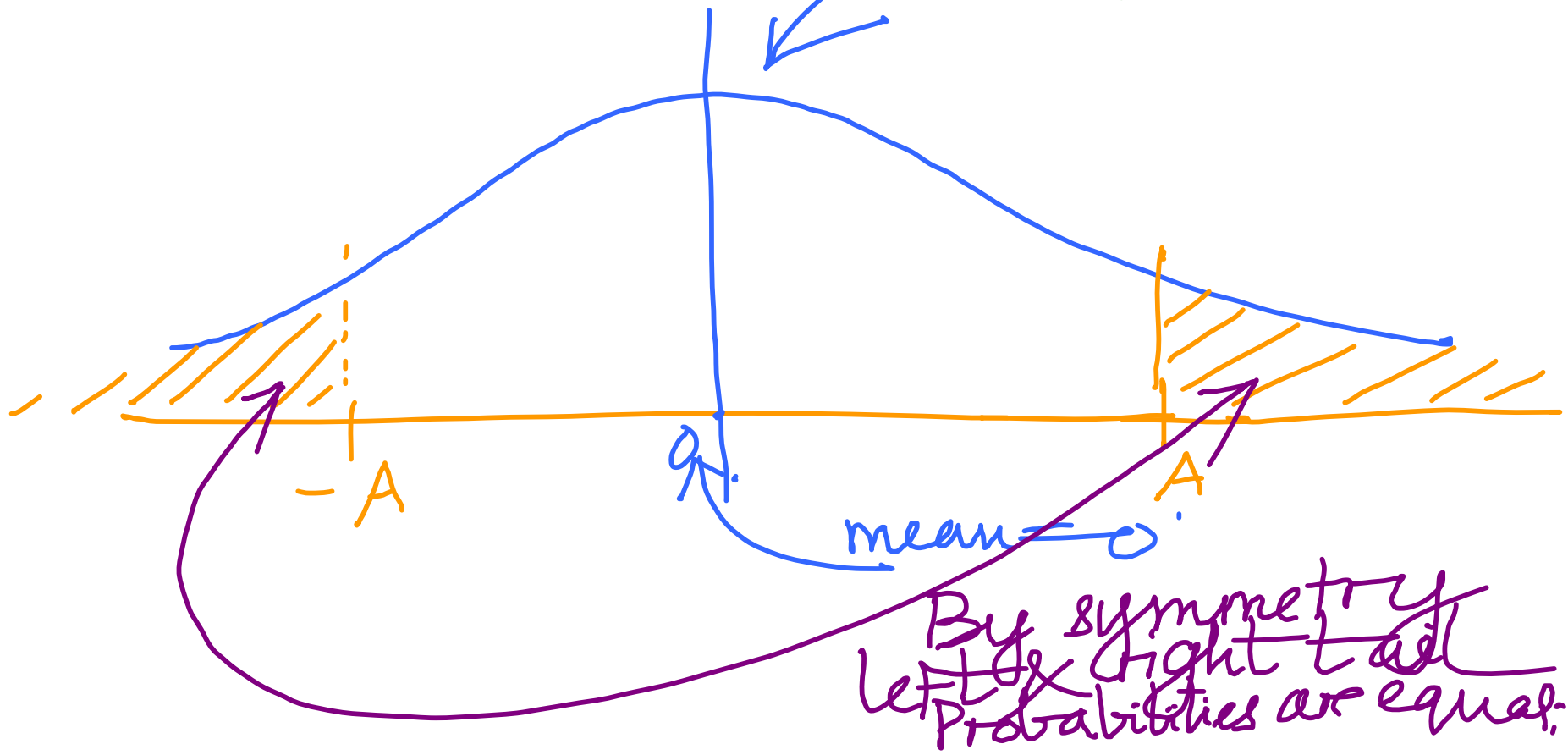
$$\Rightarrow \boxed{\tilde{n} < -A}$$

Error occurs if $\tilde{n} \geq A$
 $\tilde{n} < -A$

$$Pr(e) = P_e$$

$$= Pr(\tilde{n} \geq A) + Pr(\tilde{n} < -A)$$

PDF is symmetric about 0.



$$\Rightarrow \boxed{\Pr(\tilde{n} \geq A) = \Pr(\tilde{n} < -A)}$$

Symmetry.

$$P_e = 2 \Pr(\tilde{n} \geq A)$$

Gaussian
0 mean
var $\frac{N_0}{2} = \sigma^2$

$$= 2 Q\left(\sqrt{\frac{A^2}{N_0/2}}\right) = Q\left(\frac{A}{\sigma}\right)$$

$$= 2 Q\left(\sqrt{\frac{A^2}{N_0/2}}\right).$$

$$2 \Pr(\tilde{n} > A)$$

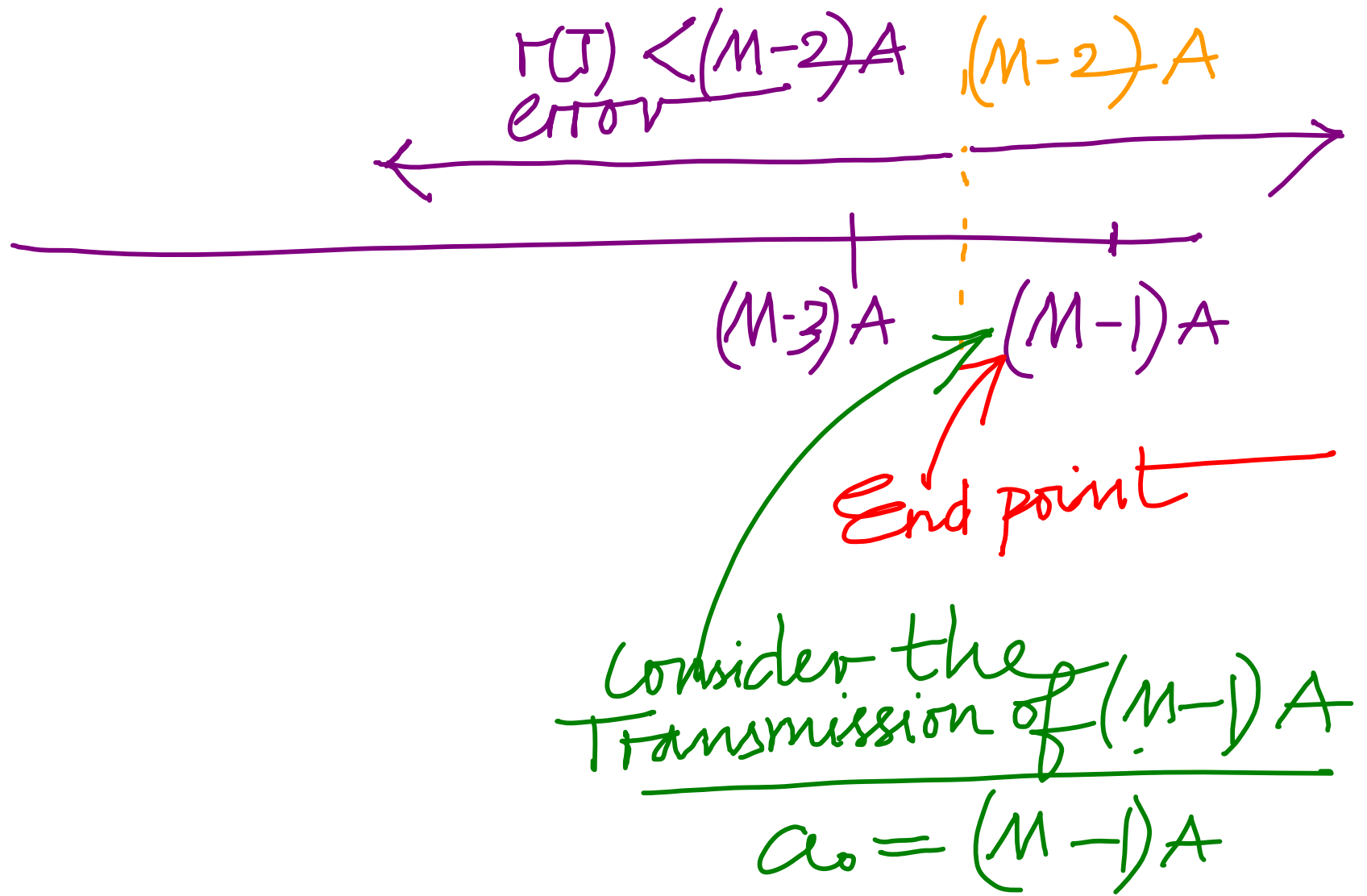
$$= 2 \Pr\left(\frac{\tilde{n}}{\sigma} > \frac{A}{\sigma}\right) = Q\left(\frac{A}{\sigma}\right).$$

Standard
Gaussian
mean = 0
var = 1

internal
point

$$Q(x) = \Pr(\text{Std Gauss.} > x)$$

$$P_e = 2Q\left(\sqrt{\frac{A^2}{N_0/2}}\right).$$



$$\Rightarrow r(T) = (m-1)A + \tilde{n}$$

$$\text{Error if } (m-1)A + \tilde{n} < (m-2)A$$

$$\Rightarrow \tilde{n} < -A$$

$$P_e = P(\tilde{n} < -A)$$

From symmetry

$$= P(\tilde{n} > A)$$

$$= P\left(\frac{\tilde{n}}{\sigma} > \frac{A}{\sigma}\right)$$

Endpoints.

Std Gaussian

$$P_e = Q\left(\frac{A}{\sigma}\right)$$

Assuming all constellation points are Equiprobable.

$$\Pr(a_0 = \pm(2i+1)A) = \frac{1}{M}$$

$$\Pr(\text{End point}) = \frac{2}{M}$$

$$\Pr(\text{Int Pt}) = \frac{M-2}{M}$$

Average P_e

$$= \Pr(e|\text{End}) \Pr(\text{End}) + \Pr(e|\text{Int}) \Pr(\text{Int})$$

mutually exclusive
exhaustive

$$= Q\left(\frac{A}{\sigma}\right) \cdot \frac{2}{N} + 2Q\left(\frac{A}{\sigma}\right) \frac{N-2}{N}$$

$$= \frac{2+2(N-2)}{N} \cdot Q\left(\frac{A}{\sigma}\right)$$

$$P_e = 2\left(1 - \frac{1}{N}\right) Q\left(\frac{A}{\sigma}\right)$$

$$\sigma^2 = \frac{N_0}{2}$$

$$\Rightarrow \sigma = \sqrt{\frac{N_0}{2}}$$

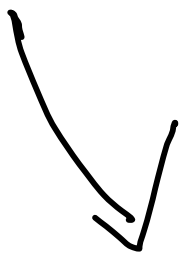
$$A = \sqrt{\frac{3E_s}{M^2 - 1}}$$

Average
Symbol
Energy

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{6E_s}{N_0(M^2 - 1)}}\right)$$

Probability of
Error for M-ary PAM

$$M = 2 \quad P_e = Q\left(\sqrt{\frac{2E_s}{N_0}}\right)$$


$$\checkmark \quad \underline{\# \text{ bits/symbol} = \log_2 M}$$

By loading large number of bits per symbol, one is able to achieve higher data rates in 4G/5G wireless systems.